How to compute E(X)?

1. Definition (requires a PMF/PDF that is easy to work with)
2. LOTUS (write X = g(Y) where Y has a PMF/PDF that is easy to work with)
   1. Also works for multivariate case: Write $X=g(Y,Z)$ where (Y,Z) has a joint PMF/PDF that’s easy to work with. But joint PDFs often harder to work with than marginal PDFs, use as a last resort
3. Independent implies uncorrelated: If X and Y are independent, g(X) and h(Y) are independent, hence uncorrelated, so E[g(X)h(Y)]=E[g(X)]E[h(Y)]
4. Linearity (write X = Y\_1 + … + Y\_n where each Y\_i has known expectation, e.g. an indicator; note the Y\_i do not have to be independent)
5. LOTE: When there’s an event/event partition that when conditioned on, makes the problem easier
   1. Bidding problem
   2. Two heads in a row: first-step analysis (LOTE)
6. Law of iterated expectations: When there’s a random variable that when conditioned on, makes the problem easier
   1. Random sum

Example problem:

Beta, variance using bank-post office

A close up of a text

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How to compute Var(X)?

1. Compute E(X^2) – (E(X))^2; for each term, use tricks above
2. Write Var(X) = Cov(X,X), then expand out using bilinearity
   1. Special case: Write X=Y\_1+…+Y\_n where Y\_1,…,Y\_n are independent, then Var(X) = Var(Y\_1) + … + Var(Y\_n)
   2. Variance of Hypergeometric
3. Law of total variance: When there’s a random variable that when conditioned on, makes the problem easier

Might simplify by remembering key properties of distributions:

* Scaling properties of Uniform, Normal, Exponential, Gamma
* X binomial, n-X binomial
* Re-centering of Uniform, Normal
* Gamma = sum of expo
* NBin = sum of geom
* Beta = Expo\_1/Expo\_1+Expo\_2

Example problems:

Joint distributions:

* 1. Conversion between joint PDF/PMF and conditional/marginal PDF/PMFs
     1. Basically just Bayes’ rule and LOTP
     2. Chicken egg
     3. Beta-binomial conjugacy
     4. Important computation trick: Drop normalizing constants, pattern match since PDFs/PMFs are uniquely defined once they are known up to a proportionality constant
  2. Change of variables
     1. Bank post-office
     2. 3 step formula:
        1. A math problem with equations

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  3. Symmetry: X,Y iid -> X,Y exchangeable, i.e. (X,Y) and (Y,X) have the same distribution. Can have exchangeability without iid.
  4. Fallback option: Double integrate the joint PDF
     1. Uniform trick
     2. P(T\_1 < T\_2) for Expo
     3. Independent: joint PDF factors

A screenshot of a math test

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